**ECE 406**

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**Assignment 1**

1. is if for all

Let

Therefore, with N=9 and c=1, is .

As n increases to large numbers, . Therefore, is .

1. * 2. It is neither one because cos(n) periodically cycles between -1 and 1. This means that g(n) cycles between and or and . There exists no constant for which as n increases and no constant for which as n increases. As a result, there is no constant that can be multiplied to g(n) to satisfy , , or .
2. I) To use induction, it will be proved that if , then .

Since then

Therefore,

Therefore, if ,

Therefore, if , .

To fully satisfy the conditions, it must be proved that and .

Therefore if .

II) Let c = 0.9. The inequality can be proved using induction. Let .

First,

Therefore, the inequality holds for . Next, it will be shown that if , then .

Let k be some n.

Since then

Therefore,

Therefore, and . Since , .

Therefore, the inequality holds true for .

III) The ratio between numbers in the Fibonacci sequence can be observed below.

|  |  |
| --- | --- |
| Number | Ratio from Previous Number () |
| 1 | - |
| 2 | 2 |
| 3 | 1.5 |
| 5 | 1.666666667 |
| 8 | 1.6 |
| 13 | 1.625 |
| 21 | 1.615384615 |
| 34 | 1.619047619 |
| 55 | 1.617647059 |
| 89 | 1.618181818 |
| 144 | 1.617977528 |
| 233 | 1.618055556 |
| 377 | 1.618025751 |
| 610 | 1.618037135 |
| 987 | 1.618032787 |
| 1597 | 1.618034448 |
| 2584 | 1.618033813 |
| 4181 | 1.618034056 |
| 6765 | 1.618033963 |
| 10946 | 1.618033999 |

The ratio is calculated as:

As shown above, the ratio approaches approximately 1.618. If continued, this approximates the golden ratio:

To determine the value of c, this ratio needs to be expressed as a power of 2.

Thus, the largest value of c for which is .

1. I) First, based on the given information, and

Therefore, since N can be factored from the first expression, the remainder of dividing by N is . In other words:

II) Induction can be used to show that . First,

Using the proof from part (I), if and , . In this case, , , , and Therefore, since every time another 9 (y’) is multiplied in, the remainder is multiplied by 1 (y), it follows the remainder remains the same for all integers.

Next, since , then can be rewritten as . This is equal to 1 no matter the value of k.

It follows that, since both expressions are equal to 1,

III) The proof in part (II) holds because x’ and y’ are larger than N; 9 > 8. To make this the case, the question can be rewritten as follows.

Using the proof from part (II) and the fact that ,

In this case, k=67 and the proof holds true.

1. Let a base b double-digit number be represented as XY, where X is the coefficient of and Y is the coefficient of . When adding three single digit base b numbers, P, Q, R,

If , then the result will be 3 digits long instead of 2 digits long. The largest value of any single-digit number, P, Q, or R, is b-1, otherwise it would become a 2-digit number. Replacing each with the largest possible value,

Since:

for any

It follows that and therefore or since X is an integer. Therefore, since , , so the result of will be at most a 2-digit number.



1. An integer x is the multiplicative inverse of if . To bring the question to this format, divide by 9:

If multiplicative inverse exists, it can be found as x in ext-Euclid(a, N). Using the function from question 6 with inputs and ,

(x, y, d) = ext-Euclid(a, N)

x = 2, y = -1, d = 1

Therefore, the first answer is . However, since , the remainder value 1 repeats every 7 numbers. Therefore,

, where k is an integer

Therefore,

Since, 4 is not divisible by 7, must be divisible by 7. Therefore,

, where b is an integer and

Therefore,